10 Conic Sections

Roller coasters are wonderful examples of parabolas as they curve in just one direction. This conic form is excellent for Roller coasters because it provides riders the feeling of climbing a hill, reaching the peak, then coasting down at high speeds.

Topic Notes

- *Circle and Parabola*
- *Ellipse and Hyperbola*

CIRCLE AND PARABOLA 1

TOPIC 1

SECTIONS OF A CONE

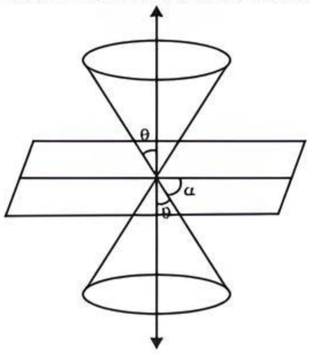
Section of a Doubled Napped Right Circular Cone by a Plane

Consider a double napped right circular cone, having semi-vertical angle θ. Let α be the angle between the plane and the axis of cone.

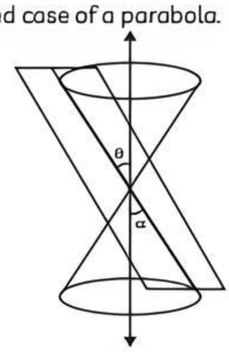
Case I: When the plane passes through vertex of the cone

If the plane passes through the vertex of the cone, then it cuts both nappies. According to the value of α, we get the following sections:

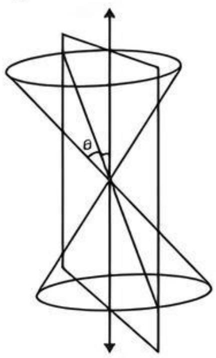
(i) When $\theta < \alpha \leq 90°$, then the section is a point.



(ii) When $\theta = \alpha$, then the plane contains the generator of cone and the section is a straight line. It is a degenerated case of a parabola.

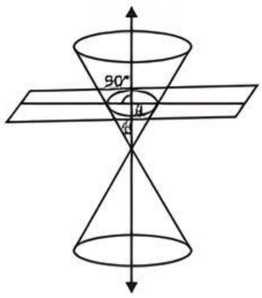


(iii) When $0 \leq \alpha < \theta$, then section is a pair of straight lines. It is a degenerated case of a hyperbola.

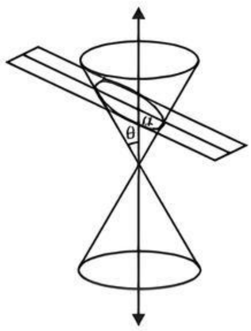


Case II: When the plane does not pass through vertex of the cone, then it cuts only one nappe. According to the value of α, we get the following sections:

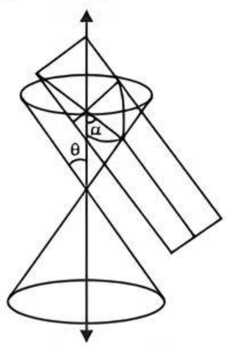
(i) When $\alpha = 90°$, then cutting plane section is a circle.



(ii) When $\theta < \alpha < 90°$, then cutting plane section is an ellipse.



(iii) When $\alpha = \theta$ i.e., plane is parallel to a generator, then cutting plane is parabola.

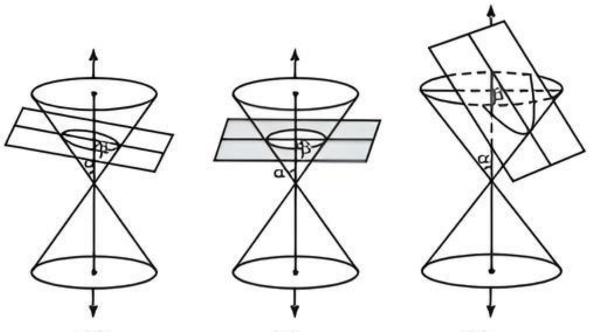


Circle, Ellipse, Parabola: When the plane cuts the nappe (other than the vertex) of the cone, we have the following situations:

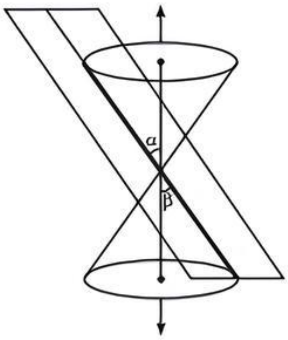
(1) When $\beta = 90°$, the section is a circle (As shown in figure (a)).

(2) When $\alpha < \beta < 90°$, the section is an ellipse (As shown in figure (b)).

(3) When $\beta = \alpha$; the section is a parabola (As shown in figure (c)).



(a) (b) (c)

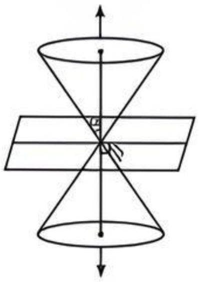


(e)

Important

➥ *Degenerated conic sections:*

(1) *When $\alpha < \beta \le 90°$, then the section is a point (As shown in figure (d))*



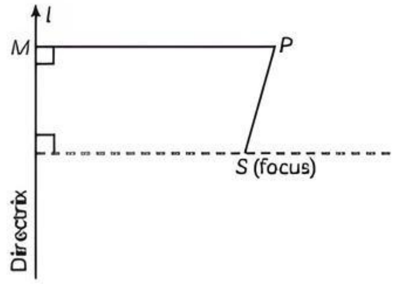
(d)

(2) *When $\beta = \alpha$, the plane contains a generator of the cone and the section is a straight line.*
It is the degenerated case of a parabola. (As shown in figure (e)).

Conic

(1) The locus of a point which moves in a plane such that the ratio of its distance from a fixed point to its perpendicular distance from a fixed straight line is always constant, is known as a conic section or a conic.

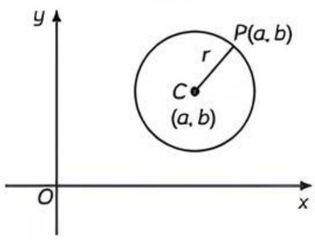
(2) The fixed point (S) is called focus of the conic, the fixed line (l) is called directrix of the conic and the constant ratio is called eccentricity of the conic.



TOPIC 2

CIRCLE

A circle is the locus of a point which moves in a plane so that its distance from a fixed point remains constant. The fixed point and the constant distance are called centre and radius of the circle, respectively.



Standard Equation of Circle

(1) If coordinates of the end points of the diameter are $(x_1, y_1), (x_2, y_2)$ then equation of circle is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ whose centre is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ and radius is,

$$\frac{\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right)}{2}.$$

(2) The equation of circle with centre (h, k) and radius a is,

$$(x - h)^2 + (y - k)^2 = a^2 \quad \text{...(i)}$$

Equation of Circle in Special Cases

If Centre Coincide with Origin

As $h = k = 0$, then equation of circle be $x^2 + y^2 = a^2$

When Circle Touches x-axis

As $C(h, k)$ be the centre of the circle and touches x-axis.

$\therefore \quad a = k$, then $(x - h)^2 + (y - k)^2 = a^2$

reduces to

$$(x - h)^2 + (y - a)^2 = a^2$$

Or, $\quad x^2 + y^2 - 2xh - 2ay + h^2 = 0$

When Circle Passes Through (0, 0)

Then the equation of circle (i) reduces to

$$(x-h)^2+(y-k)^2=h^2+k^2$$

Or, $x^2+y^2-2hx-2ky=0$

When Circle Touches y-axis

As circle touches y-axis

$\therefore$ $h=a$ and equation of circle

$$(x-h)^2+(y-k)^2=a^2$$

reduces to

$$(x-a)^2+(y-k)^2=a^2$$

$$x^2+y^2-2ax-2ky+k^2=0$$

Equation of Circle Touches Both Axis

In this case $h=k=a$ and equation of circle

$$(x-h)^2+(y-k)^2=a^2$$

reduces to

$$(x-a)^2+(y-a)^2=a^2$$

Or $x^2+y^2-2a(x+y)=-a^2$

Equation of Circle Passing Through (0, 0) and Centred at x-axis

Then $k=0$, $h=a$ and equation $(x-h)^2+(y-k)^2=a^2$ reduces to $x^2+y^2-2ax=0$

Equation of Circle Passes Through (0, 0) and Centred at y-axis

In this case $h=0$, $k=a$ then the equation $(x-h)^2+(y-k)^2=a^2$ reduces to $x^2+y^2-2ay=0$

Parametric form of Equation of Circle Centreed at Origin

$$x=a\cos\theta,\ y=a\sin\theta$$

Example 1.1: Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3). [NCERT]

Ans. Let the equation of the required circle be,

$$(x-h)^2+(y-k)^2=r^2.$$

Since, the radius of the circle is 5 and its centre lies on the x-axis i.e., $k=0$ and $r=5$.

Now, the equation of the circle becomes

$$(x-h)^2+y^2=25. \quad \text{...(i)}$$

It is given that the circle passes through the point (2, 3).

$\therefore$ $(2-h)^2+(3-0)^2=25$

$\Rightarrow$ $(2-h)^2=25-9$

$\Rightarrow$ $(2-h)^2=16$

$\Rightarrow$ $2-h=\pm\sqrt{16}\pm 4$

If $2-h=4$, then $h=-2$.

If $2-h=-4$, then $h=6$.

When $h=-2$, then the equation of the circle is,

$$(x+2)^2+y^2=25$$

$$x^2+4x+4+y^2=25$$

$$x^2+y^2+4x-21=0$$

When $h=6$, then the equation of the circle is,

$$(x-6)^2+y^2=25$$

$$x^2-12x+36+y^2=25$$

$$x^2+y^2-12x+11=0$$

Hence, the equation of circle are,

$$x^2+y^2-12x+11=0$$

and $x^2+y^2+4x-21=0$

Example 1.2: Find the equation of the circle with centre $\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $\frac{1}{12}$. [NCERT]

Ans. The equation of a circle with centre (h, k) and radius r is given as $(x-h)^2+(y-k)^2=r^2$.

It is given that centre $(h, k)=\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $(r)=\frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$

$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$

$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$

$$144x^2-144x+36+144y^2-72y+9-1=0$$

$$144x^2-144x+144y^2-72y+44=0$$

$$36x^2-36x+36y^2-18y+11=0$$

$$36x^2+36y^2-36x-18y+11=0$$

Example 1.3: Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$. [NCERT]

Ans. The equation of a circle with centre (h, k) and radius r is given as

$$(x-h)^2+(y-k)^2=r^2.$$

It is given that centre $(h, k)=(1, 1)$ and radius $\sqrt{2}$.

Therefore, the equation of the circle is,

$$(x-1)^2+(y-1)^2=(\sqrt{2})^2$$

$$x^2-2x+1+y^2-2y+1=2$$

$$x^2+y^2-2x-2y=0$$

Example 1.4: Find the centre and radius of the circle $x^2+y^2-8x+10y-12=0$. [NCERT]

Ans. The equation of the given circle is,

$$x^2+y^2-8x+10y-12=0$$

$\Rightarrow$ $(x^2-8x)+(y^2+10y)=12$

$\Rightarrow$ $\{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$

$\Rightarrow$ $(x-4)^2 + (y+5)^2 = 53$

$\Rightarrow$ $(x-4)^2 + \{y-(-5)\}^2 = (\sqrt{53})^2$

Which is of the form $(x-h)^2 + (y-k)^2 = r^2$,

where $h = 4$, $k = -5$ and $r = \sqrt{53}$

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

Example 1.5: **Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$.** [NCERT]

Ans. The equation of the given circle is

$$2x^2 + 2y^2 - x = 0$$

$\Rightarrow$ $(2x^2 - x) + 2y^2 = 0$

$\Rightarrow$ $2\left[\left(x^2 - \frac{x}{2}\right) + y^2\right] = 0$

$\Rightarrow$ $\left\{x^2 - 2.x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2\right\} + y^2 - \left(\frac{1}{4}\right)^2 = 0$

$\Rightarrow$ $\left(x - \frac{1}{4}\right)^2 + (y-0)^2 = \left(\frac{1}{4}\right)^2$

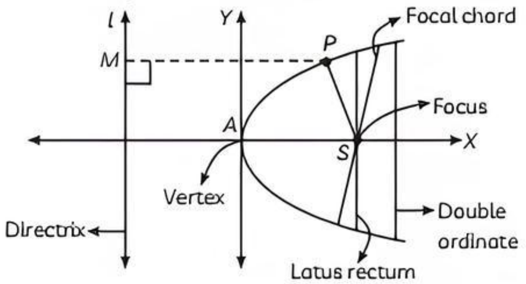
Which is of the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$ and $r = \frac{1}{4}$.

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0\right)$, while its radius is $\frac{1}{4}$.

TOPIC 3

PARABOLA

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point. The fixed line is called the directrix and the fixed point is called the focus of the parabola.



In conic section, a parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane. The fixed line is called the directrix, and the fixed point (F) is called the parabola's focus. A line through the focus and perpendicular to the directrix is called the axis of the parabola. Also, the vertex is the point of intersection of a parabola with the axis. As we know, the standard equation of the parabola is used in solving a variety of problems in maths.

In this article, you will learn how to write the standard equations for parabola in different cases and how to solve questions based on these equations.

$y^2 = 4ax$, $y^2 = -4ax$, $x^2 = 4ay$, $x^2 = -4ay$ as the standard equation of parabola.

Latus Rectum

In the conic section, the latus rectum is the chord through the focus, and parallel to the directrix. The word latus rectum is derived from the Latin word "latus" which means "side", and the "rectum" which means "straight". Half the latus rectum is called the semi latus rectum. The diagram above shows the latus rectum of a parabola.

Length of Latus Rectum of Parabola

Let the ends of the latus rectum of the parabola, $y^2 = 4ax$ be L and L'. The x-coordinates of L and L' are equal to 'a' as $S = (a, 0)$

Assume that, $L = (a, b)$.

We know that L is a point of the parabola, we have

$$b^2 = 4a(a) = 4a^2$$

Take square root on both sides, we get $b = \pm 2a$

Therefore, the ends of the latus rectum of a parabola are $L = (a, 2a)$ and $L' = (a, -2a)$

Hence, the length of the latus rectum of a parabola, 'LL'' is $4a$.

Caution

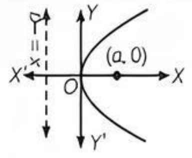
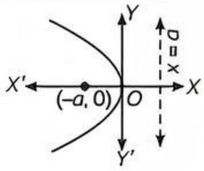
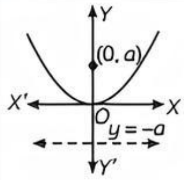
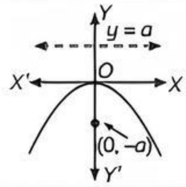
➥ *The standard equations of parabolas have focus on one of the coordinate axes; the vertex at the origin and thereby the directrix is parallel to the other coordinate axis.*

Standard Equation of Parabola

If a parabola has a horizontal axis, the standard form of the equation of the parabola is this:

$$(y-k)^2 = 4p(x-h), \text{ where } p \neq 0.$$

The vertex of this parabola is at (h, k). The focus is at $(h+p, k)$.

Standard equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Graph				
Eccentricity	$e = 1$	$e = 1$	$e = 1$	$e = 1$
Coordination of focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Coordination of vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Extremities of latus rectum	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$
Equation of latus rectum	$x - a = 0$	$x + a = 0$	$y - a = 0$	$y + a = 0$

Example 1.6: Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) passing through (2, 3) and axis is along x-axis. [NCERT]

Ans. Since, the vertex is (0, 0) and the axis of the parabola is the x-axis, the equation of the parabola is either of the form

$$y^2 = 4ax \text{ or } y^2 = -4ax.$$

The parabola passes through point (2, 3), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point (2, 3) must satisfy the equation $y^2 = 4ax$.

$\therefore \quad 3^2 = 4a(2)$

$\Rightarrow \quad a = \frac{9}{8}$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$y^2 = \frac{9}{2}x$$

$$2y^2 = 9x$$

which is the required equation of parabola.

Example 1.7: Find the equation of the parabola that satisfies the following condition: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis. [NCERT]

Ans. Since, the vertex is (0, 0) and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form

$$x^2 = 4ay \text{ or } x^2 = -4ay.$$

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point (5, 2) must satisfy the equation $x^2 = 4ay$.

$\therefore \quad (5)^2 = 4 \times a \times 2$

$\Rightarrow \quad 25 = 8a$

$\Rightarrow \quad a = \frac{25}{8}$

Thus, the equation of the parabola is,

$$x^2 = 4\left(\frac{25}{8}\right)y$$

$$x^2 = \frac{25}{2}y$$

$$2x^2 = 25y$$

Example 1.8: Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$. [NCERT]

Ans. The given equation is $y^2 = 12x$

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with, $y^2 = 4ax$, we obtain

$$4a = 12$$

$\Rightarrow \quad a = 3$

$\therefore$ Coordinates of the focus $= (a, 0) = (3, 0)$

Since the given equation involves, y^2.

The axis of the parabola is the x-axis.

Equation of directrix,

$$x = -a$$

i.e., $\quad x = -3$

i.e., $\quad x + 3 = 0$

Length of latus rectum $= 4a = 4 \times 3 = 12$.

Example 1.9: Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$. [NCERT]

Ans. The given equation is $x^2 = 6y$.

Here, the coefficient of y is positive. Hence, the parabola opens upwards.

On comparing this equation with, $x^2 = 4ay$, we obtain

$$4a = 6$$

$$\Rightarrow \quad a = \frac{3}{2}$$

$\therefore$ Coordinates of the focus $= (0, a) = \left(0, \frac{3}{2}\right)$

Since, the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$ i.e., $y = -\frac{3}{2}$

Length of latus rectum $= 4a = 6$

Example 1.10: Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) focus (–2, 0). [NCERT]

Ans. Vertex (0, 0); focus (–2, 0)

Since, the vertex of the parabola is (0, 0) and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$. Since, the focus is (–2, 0), $a = 2$.

Thus, the equation of the parabola is

$$y^2 = -4(2)x,$$

i.e., $$y^2 = -8x$$

Example 1.11: Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0); focus (3, 0). [NCERT]

Ans. Vertex (0, 0); focus (3, 0)

Since, the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis, x-axis is the axis of the parabola while the equation of the parabola is of the form $y^2 = 4ax$.

Since, the focus is (3, 0), $a = 3$.

Thus, the equation of the parabola is

$$y^2 = 4 \times 3 \times x, \text{ i.e., } y^2 = 12x$$

Example 1.12: Find the equation of the parabola that satisfies the following conditions: Focus (0, – 3); directrix $y = 3$. [NCERT]

Ans. Focus = (0, –3); directrix $y = 3$

Since, the focus lies on the y-axis, the y-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

It is also seen that the directrix, $y = 3$ is above the x-axis, while the focus (0, – 3) is below the x-axis.

Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

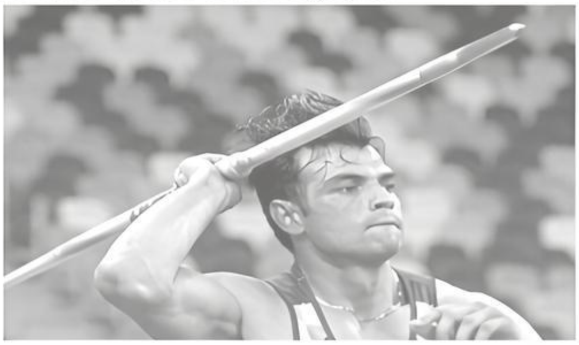
Thus, the equation of the parabola is

$$x^2 = -4 \times 3 \times y$$

$$x^2 = -12y$$

Example 1.13: Case Based:

Neeraj Chopra an athlete won a gold medal in Javelin throw, He is the first athlete who won gold medal in Javelin throw from India at Olympics.



Based on the above information, answer the following questions.

(A) Assertion (A): The shape of the path followed by a Javelin is a straight line.

Reason (R): In a game of Javelin throw, Javelin travels a certain distance and then falls down following a parablic path.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(B) If the equation of a curve is given by $y^2 = -24x$, the coordinate of the focii are:

(a) (6, 0) (b) (0, 6)

(c) (0, – 6) (d) (–6, 0)

(C) The equation of direction of parabola $y^2 - 24x$ is:

(a) $y - 6 = 0$ (b) $y + 6 = 0$

(c) $x + 6 = 0$ (d) $x - 6 = 0$

(D) Find the length of latus rectum of parabola $y^2 - 24x$ is.

(E) Find the length of the latus rectum of $x^2 = -9y$.

Ans. (A) *(d) (A) is false but (R) is true.*

Explanation: The path followed by Javelin is parabolic.

(B) *(d) (–6, 0)*

Explanation: Here, $y^2 = -24x$

Equation of parabola is $y^2 = -4ax$

So, $4a = 24$ or, $a = 6$

Here, coordinate of focii $= (-a, 0) = (-6, 0)$

(C) *(d)* $x - 6 = 0$

Explanation: If $a = 6$ for $y^2 = -4ax$ then Equation of directrix will $x - a = 0$

i.e., $x - 6 = 0$

(D) *(d)* 24

Explanation: Length of latus rectum for $y^2 = -4ax$

Here, $y^2 = 24x$

$\Rightarrow$ $4a = 24$

(E) Given parabola equation:

$$x^2 = -9y \quad ...(i)$$

Since, the coefficient of y is negative, the parabola opens downwards.

The general equation of parabola is,

$$x^2 = -4ay \quad ...(ii)$$

Comparing (i) and (ii), we get

$$-4a = -9$$

$$a = \frac{9}{4}$$

We know that the length of latus rectum

$$= 4a = 4\left(\frac{9}{4}\right) = 9.$$

Therefore, the length of the latus rectum of $x^2 = -9y$ is equal to 9 units.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Equation of a circle whose centre is (–1.5, –2.6) and radius is 5, is:

(a) $(x - 1.5)^2 + (y + 2.6)^2 = 5$

(b) $(x + 1.5)^2 + (y - 2.6)^2 = 25$

(c) $(x + 1.5)^2 + (y + 2.6)^2 = 25$

(d) none

Ans. *(c)* $(x + 1.5)^2 + (y + 2.6)^2 = 25$

Explanation: Equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

Here, $h = -1.5$, $k = -2.6$ and $r = 5$

So,

equation will be

$$(x + 1.5)^2 + (y + 2.6)^2 = 25$$

2. If the parabola passes through the point (–3, –2) then the length of its latus rectum is, if equation is $y^2 = 4ax$:

(a) $-\frac{2}{3}$ (b) $-\frac{4}{3}$

(c) $-\frac{1}{3}$ (d) -4

Ans. *(b)* $-\frac{4}{3}$

Explanation: Here, equation of parabola is given by

$$y^2 = 4ax$$

Since, it passes through (–3, –2), we get

On putting $x = -3$ and $y = -2$

$$4 = -12a$$

$$-\frac{1}{3} = a$$

$\therefore$ The length of latus rectum $= 4a$

$$= 4 \times -\frac{1}{3} = -\frac{4}{3}$$

3. Equation of a circle whose centre is (0, 0) and radius r is:

(a) $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = r^2$

(c) $y^2 - x^2 = r^2$ (d) none

Ans. *(b)* $x^2 + y^2 = r^2$

Explanation: We know that equation of circle is given by,

$$(x - h)^2 + (y - k)^2 = r$$

Given, centre $(h, k) = (0, 0)$ and radius $= r$

$$h = 0, k = 0, r = r$$

So equation will be

$$x^2 + y^2 = r^2$$

4. Equation of a circle with centre (–a, –b) and radius = 100 is:

(a) $(x - a)^2 + (y - b)^2 = 100$

(b) $(x + a)^2 + (y + b)^2 = 10000$

(c) $x^2 + y^2 = 100$

(d) None

Ans. *(b)* $(x + a)^2 + (y + b)^2 = 10000$

Explanation: We know that the equation of a circle is given by,

$$(x - h)^2 + (y - k)^2 = r^2$$

Here, $h = -a$, $k = -b$ and $r = 100$

So equation will be,

$$(x + a)^2 + (y + b)^2 = 10000$$

5. Latus rectum of the parabola $y^2 = 8x$ is:

(a) 2 (b) 4

(c) 6 (d) 8 [Diksha]

Ans. *(d)* 8

Explanation: Equation of parabola is $y^2 = 8x$

On comparing it with $y^2 = 4ax$, we get

$$4a = 8$$

We know that length of latus rectum $= 4a = 8$

6. Equation of circle with centre (2, –3) and radius 8 is:

(a) $x^2 + y^2 - 4x + 6y - 51 = 0$
(b) $x^2 + y^2 - 4x - 6y + 51 = 0$
(c) $x^2 + y^2 + 4x - 6y - 51 = 0$
(d) $x^2 + y^2 - 4x + 6y + 51 = 0$ [Diksha]

Ans. *(a)* $x^2 + y^2 - 4x + 6y - 51 = 0$

Explanation: Equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

Here, $h = 2$, $k = -3$ and $r = 8$

So, equation will be,

$$(x - 2)^2 + (y + 3)^2 = 64$$
$$x^2 + 4 - 4x + y^2 + 9 + 6y = 64$$
$$x^2 + y^2 - 4x + 6y - 51 = 0$$

7. The value of eccentricity for the parabola is:

(a) 1
(b) less than 1
(c) more than 1
(d) don't have eccentricity [Diksha]

Ans. *(a)* 1

Explanation: The eccentricity of parabola is 1.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

8. Assertion (A): A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.

Reason (R): The point of intersection of parabola with axis is called the vertex of the parabola.

Ans. *(d) (A) is false but (R) is true.*

Explanation: A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of the parabola with the axis is called the vertex of the parabola.

9. Parabola is symmetric with respect to the axis of the parabola.

Assertion (A): If the equation of standard parabola has a term y^2, then the axis of symmetry is along the x-axis.

Reason (R): If the equation of standard parabola has a term x^2, then the axis of symmetry is along the x-axis.

Ans. *(c) (A) is true but (R) is false.*

Explanation: If the equation has a term y^2, then the axis of symmetry is along the x-axis and if the equation has a term x^2, then the axis of symmetry is along the y-axis.

10. Assertion (A): The eccentricity of a parabola is 1.

Reason (R): The eccentricity of a circle is greater than 1.

Ans. *(c) (A) is true but (R) is false.*

Explanation: The eccentricity of a parabola is exactly 1.

The eccentricity of a circle is 0.

11. Assertion (A): The parabola $y^2 = 8x$ where the value of a is 2.

Reason (R): The equation of a circle $x^2 + y^2 = 25$ having centre on + ve x-axis and radius is 5.

Ans. *(c) (A) is true but (R) is false.*

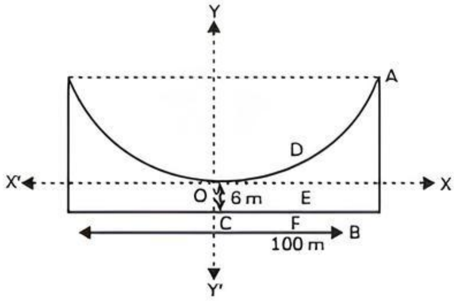
Explanation: The parabola $y^2 = 8x$, here $4a = 8$ or, $a = 2$

The equation of circle $x^2 + y^2 = 25$ having centre centre (0, 0) means at origin and radius is 5.

CASE BASED Questions (CBQs)

[4 & 5 marks]

12. *The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway, which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. A supporting wire is also attached to the roadway 18 meter from the middle. (see figure below)*



Based on the above information, answer the following questions.

(A) Find the value of a in the standard equation.

(a) $\frac{24}{625}$ (b) $\frac{18}{625}$

(c) $\frac{625}{24}$ (d) $\frac{625}{18}$

(B) Find the length of a supporting wire attached to the roadway 18 m from the middle.

(a) 3.11 m (b) 6 m

(c) 9.11 m (d) none of these

(C) What are the coordinates of point A?

(a) (50, 30) (b) (50, 24)

(c) (30, 50) (d) (24, 50)

(D) What is a particular equation of parabola?

(a) $x^2 = 25y$ (b) $x^2 = 24y$

(c) $x^2 = 625y$ (d) $6x^2 = 625y$

(E) What is the standard equation of parabola in this case?

(a) $y^2 = 4ax$ (b) $y^2 = -4ax$

(c) $x^2 = 4ay$ (d) $x^2 = -4ay$

Ans. (A) (c) $\frac{625}{24}$

Explanation: Since, $A(50, 24)$ is a point on the parabola.

$$(50)^2 = 4a(24)$$

$$\Rightarrow \quad a = 50\times\frac{50}{4}\times24$$

$$= \frac{625}{24}$$

(B) (c) 9.11 m

Explanation: The x-coordinate of point D is 18.

Hence, at $x = 18$

$$6(18)^2 = 625y$$

$$\Rightarrow \quad y = 6\times18\times\frac{18}{625}$$

$$\Rightarrow \quad y = 3.11 \text{ (approx.)}$$

$$\therefore \quad DE = 3.11 \text{ m}$$

$$DF = DE + EF$$

$$= 3.11 \text{ m} + 6 \text{ m}$$

$$= 9.11 \text{ m}$$

(C) (b) (50, 24)

Explanation: Here, AB = 30 m, OC = 6 m, and $BC = \frac{100}{2} = 50$ m

The coordinates of point A are (50, 30 −6) = (50, 24)

(D) (d) $6x^2 = 625y$

Explanation: Equation of the parabola.

$$x = 4\times\frac{625}{24}\times y$$

or $$6x^2 = 625y$$

(E) (c) $x^2 = 4ay$

Explanation: The equation of the parabola is of the form $x^2 = 4ay$ (as it is opening upwards).

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

13. Find the centre and radius of the circle whose equation is $(x-\sqrt{7})^2+(y-\sqrt{3})^2=48$.

Ans. As we know that general equation of circle is

$$(x-h)^2+(y-k)^2=r^2$$

Here, (h, k) is centre and radius is r.

Given, equation of a circle is

$$(x-\sqrt{7})^2+(y-\sqrt{3})^2=(\sqrt{48})^2$$

On comparing it with $(x-h)^2+(y-k)^2=r^2$, we get

Centre = $(\sqrt{7}, \sqrt{3})$ and radius = $\sqrt{48}$

14. Find the equation of parabola whose vertex is (0, 0) and focus is $(\sqrt{3}, 0)$.

Ans. Equation of parabola is $y^2 = 4ax$

Here, $a = \sqrt{3}$

So, equation will be,

$$y^2 = 4\sqrt{3}x$$

15. Find the coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4. [NCERT Exemplar]

Ans. Given parabola is $y^2 = 8x$ ...(i)

We know that equation of parabola is

$$y^2 = 4ax \quad ...(ii)$$

From eq. (i) and (ii)

$$8x = 4ax$$

$\Rightarrow$ $a = 2$

$\therefore$ Focal distance $= |x + a| = 4$

$\Rightarrow$ $|x + 2| = 4$

$\Rightarrow$ $x + 2 = \pm 4$

$\Rightarrow$ $x = 2, -6$

But $x \neq -6$

For $x = 2,$

$y^2 = 8 \times 2$

$\therefore$ $y^2 = 16$

$\Rightarrow$ $y = \pm 4$

So, the points are (2, 4) and (2, –4).

16. If parabola $y^2 = px$ passes through point (2, –3) then, find the length of latus rectum. **[Delhi Gov. QB 2022]**

Ans. Since the point (–2, 3) is on the parabola $y^2 = px$, we must have:

$(3)^2 = p(-2)$

$\Rightarrow$ $p = -4.5$

Comparing this with the standard equation $y^2 = 4ax$, we can say that the length of the latus rectum is $4a = p$.

As we know, length cannot be negative so the length of latus rectum $= 4a = |p| = 4.5$.

17. Find the equation of the circle having (1, –2) as its centre and passing through $3x + y = 14$, $2x + 5y = 18$. **[NCERT Exemplar]**

Ans. Given that, centre of the circle is (1, – 2) and the circle passing through the lines

$3x + y = 14$ ...(i)

And $2x + 5y = 18$ ...(ii)

Solving eq. (i) and (ii), we get $x = 4$ and $y = 2$

$\therefore$ Radius of the circle $= \sqrt{(4-1)^2+(2+2)^2}$

$= \sqrt{9+16} = 5$

Now, equation of the circle is $C(1, -2)$ and radius 5.

$(x - 1)^2 + (y + 2)^2 = 5^2$

$\Rightarrow x^2 - 2x + 1 + y^2 + 4y + 4 = 25$

$\Rightarrow x^2 + y^2 - 2x + 4y - 20 = 0$

18. Find the vertex of the parabola $4x^2 + y = 0$.

Ans. Given, $4x^2 + y = 0$

$4x^2 = -y$

$$x^2 = \frac{-1}{4}y$$

We know that the equation of circle is

$x^2 = 4ay$

Here, $4a = \frac{-1}{4}$

$$a = \frac{-1}{16}$$

So, the vertex will be = (0, 0)

19. Find the equation of the parabola with vertex at (0, 0) and focus at (0, –2.5).

Ans. Equation of the parabola is

$x^2 = -4ay$

$\Rightarrow$ $x^2 = -4(2.5)y$ $\quad [\because$ focus $(0, -a) = (0, -2.5)]$

$\Rightarrow$ $x^2 = -10y$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

20. Find the co-ordinate of focus, and length of latus rectum of parabola $10y^2 = -60x$.

Ans. Given, equation of the parabola is,

$10y^2 = -60x$

$$\Rightarrow \quad y^2 = -\frac{60}{10}x$$

$$\Rightarrow \quad y^2 = -4.\frac{3}{2}.x$$

On comparing it with $y^2 = -4ax$, we get

$$a = \frac{3}{2}$$

$$\therefore \quad \text{focus} = (-a, 0) = \left(-\frac{3}{2}, 0\right)$$

And length of latus rectum $= 4a = 4 \times \frac{-3}{2} = -6$.

21. Find the equation of the circle which passes through the origin and cuts off intercepts 3 and 4 from the positive parts of the axes.

Ans. Here, passing points are (3, 0) and (0, 4) and centre is at (0, 0)

So, the equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

For (3, 0)

$(3)^2 + (0)^2 + 2g(3) + 2f(0) + 0 = 0$

$9 + 6g = 0$

$$\Rightarrow \quad g = \frac{-3}{2}$$

For (0, 4)

$(0)^2 + (4)^2 + 2g(0) + 2f(4) + 0 = 0$

$16 + 8f = 0$

$\Rightarrow \quad f = -2$

We know that the equations of circle which passes through the origin is given by

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore \quad x^2 + y^2 - 3x - 4y = 0$

22. If a circle passes through the points (0, 0), (*a*, 0) and (0, *b*), then find the co-ordinates of its centre. [NCERT Exemplar]

Ans. Let, the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, this circle passes through the points $A(0, 0)$, $B(a, 0)$ and $C(0, b)$.

Putting (0, 0) we get $c = 0$

Putting $(a, 0)$ we get $a^2 + 2ag = 0$ ...(ii)

and putting $(0, b)$ we get $b^2 + 2bf = 0$ ...(iii)

From eq.(ii), $a + 2g = 0$

$\Rightarrow \quad g = -\dfrac{a}{2}$

From eq.(iii), $b + 2f = 0$

$\Rightarrow \quad f = -\dfrac{b}{2}$

$\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$ is the centre of the circle.

23. Prove that the points (1, 2), (3, –4),(5, –6) and (11, – 8) are concyclic. [NCERT Exemplar]

Ans. Let, the equation of the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

We get the equation of circle passing through (1, 2), (3, –4) and (5, –6) as

$x^2 + y^2 - 22x - 4y + 25 = 0$ ...(ii)

Also, point (11, – 8) lies on (ii).

$\because \quad x^2 + y^2 - 22x - 4y + 25 = 121 + 64 - 242 + 32 + 25 = 0$

Hence, the given points are concyclic.

24. Find the equation of the circle which passes through the points (1, –2) and (4, –3) has its centre on the line 3*x* + 4*y* = 7. [Diksha]

Ans. Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since it passes through (1, –2) and (4, –3)

$\therefore \quad 5 + 2g - 4f + c = 0$ ...(ii)

and $\quad 25 + 8g - 6f + c = 0$ ...(iii)

Also $(-g, -f)$ centre of circle (i) lies on $3x + 4y = 7$

$-3g - 4f = 7$ ...(iv)

Subtracting eq. (ii) from (iii), we get

$20 + 6g - 2f = 0$ ...(v)

Solving eq. (iv) and (v), we get

$g = -\dfrac{47}{15}, f = \dfrac{3}{5}\left(= \dfrac{9}{15}\right)$

Substituting these values of g and f in eq. (ii), we get

$c = \dfrac{11}{3}\left(= \dfrac{55}{15}\right)$

$\therefore$ From eq. (i), we get

$x^2 + y^2 + \dfrac{94}{15}x + \dfrac{18}{15}y + \dfrac{55}{15} = 0$

$15x + 15y - 94x + 18y + 55 = 0$

which is the required equation on the circle.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

25. Find the value of *k* such that the line *x* cos θ + *y* sin θ – *k* = 0 touches the circle $x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = 0$.

Ans. Given equation of circle is,

$x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = 0$

Which represents a circle (in general form) with centre $(a\cos\theta, a\sin\theta)$

and radius $\sqrt{a^2\cos^2\theta + a^2\sin^2\theta - 0} = a$

Given that, the line $x\cos\theta + y\sin\theta - k = 0$ touches the given circle.

So, the perpendicular distance of the line form $(a\cos\theta, a\sin\theta)$ is equal to the radius r of the circle.

$\therefore \quad r = \left|\dfrac{(a\cos\theta)\cos\theta + (a\sin\theta)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$\Rightarrow \quad a = |a - k|$

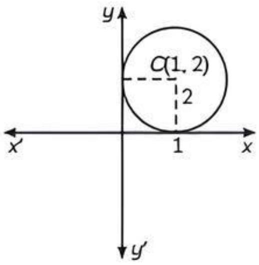
$\Rightarrow \quad \pm a = a - k$

$\Rightarrow \quad k = 0, 2a.$

26. Find the equation of the circle which touches *x*-axis and whose centre is (1, 2).

Ans. Given that, centre of the circle is (1, 2).

$(x - h)^2 + (y - k)^2 = r^2$



Put, $\quad h = 1, k = 2$

$\because \quad r = 2$

So, the equation of circle is

$$(x-1)^2 + (y-2)^2 = 2^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$

$$\Rightarrow \quad x^2 - 2x + y^2 - 4y + 1 = 0$$

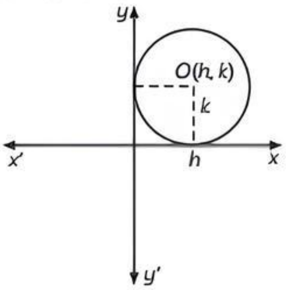
$$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$$

27. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a. **[NCERT Exemplar]**

Ans. Given that radius of the circle is a i.e., (h, k)

$$(x-h)^2 + (y-x)^2 = r^2$$

Put, $h = k = r = a$



So, the equation of required circle is

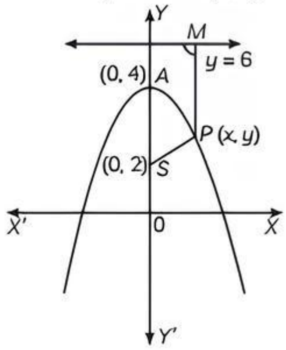
$$(x-a)^2 + (y-a)^2 = a^2$$

$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = a^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay + a^2 = 0$$

28. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola. **[NCERT Exemplar]**

Ans. Given that the coordinates, vertex of the parabola (0, 4) and focus of the parabola (0, 2).



$$PM = PS.$$

$$\Rightarrow \quad \left|\frac{0+y-6}{\sqrt{0+1}}\right| = \sqrt{(x-0)^2 + (y-2)^2}$$

$$\Rightarrow \quad |y-6| = \sqrt{x^2 + y^2 - 4y + 4}$$

$$\Rightarrow \quad x^2 + y^2 - 4y + 4 = y^2 + 36 - 12y$$

$$\Rightarrow \quad x^2 + 8y = 32$$

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

29. Find the equation of the parabola whose vertex is the point (0, 2) and the directrix is the line $x + y - 3 = 0$.

Ans. Given, vertex of parabola is (0, 2) and directrix of parabola is,

$$x + y - 3 = 0 \quad \text{...(i)}$$

Then, slope of directrix $= -1 = m_1$ (say)

Since, axis of parabola is perpendicular to the directrix.

So, slope of axis $= -\dfrac{1}{m_1} = 1 = m_2$ (say)

Then, the equation of axis is of the form

$$y = m_2 x + c,$$

i.e., $\quad y = x + c.$

Since, axis passes through the vertex (0, 2).

$\therefore \quad 2 = 0 + c$

$\Rightarrow \quad c = 2$

So, the equation of axis of parabola is,

$$y = x + 2. \quad \text{...(ii)}$$

On solving (i) and (ii), we get $x = \dfrac{1}{2}, y = \dfrac{5}{2}$

The point of intersection of the directrix and axis is $D\left(\dfrac{1}{2}, \dfrac{5}{2}\right)$.

Let, $F(\alpha, \beta)$ be the coordinates of the focus.

The mid-point of FD is $\left(\dfrac{\alpha + \frac{1}{2}}{2}, \dfrac{\beta + \frac{5}{2}}{2}\right)$

We know that the vertex V is the mid-point of FD.

$$\therefore \quad \frac{\alpha + \frac{1}{2}}{2} = 0 \text{ and } \frac{\beta + \frac{5}{2}}{2} = 2$$

i.e., $\alpha = -\frac{1}{2}$ and $\beta = \frac{3}{2}$.

So, the coordinates of focus of parabola are $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

Let, $P(x, y)$ be any point on the parabola.

Then, by definition of parabola, we have

Distance of $P(x, y)$ from $\left(-\frac{1}{2}, \frac{3}{2}\right)$ =

Perpendicular distance of $P(x, y)$ from $x + y - 3 = 0$

$$\Rightarrow \sqrt{\left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2} = \left|\frac{x+y-3}{\sqrt{(1)^2+(1)^2}}\right|$$

$$\Rightarrow \sqrt{\left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2} = \left|\frac{x+y-3}{\sqrt{2}}\right|$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2+\left(y-\frac{3}{2}\right)^2 = \frac{(x+y-3)^2}{2}$$

$$\Rightarrow 2\left(x^2+x+\frac{1}{4}\right)+y^2-3y+\frac{9}{4}$$

$$= x^2 + y^2 + 9 + 2xy - 6x - 6y$$

$$\Rightarrow x^2 - 2xy + y^2 + 8x - 4 = 0$$

which is the required equation of parabola.

30. Find the coordinates of focus, axis of parabola, the equation of directrix, coordinate of vertex of the following parabola

$$\mathbf{x^2 - 2x - 4y - 11 = 0.}$$

Ans. Given, equation of the parabola is,

$$x^2 - 2x - 4y - 11 = 0$$

$$\Rightarrow (x^2 - 6x + 4) - 4 - 4y - 11 = 0$$

$$\Rightarrow (x-2)^2 = 4y + 20$$

$$\Rightarrow (x-2)^2 = 4(y+5)$$

$$= 4 \times 1 \times (y+5)$$

For vertex,

$$\therefore x - 2 = 0$$

$$\Rightarrow x = 2$$

$$\therefore y + 5 = 0$$

$$\Rightarrow y = -5$$

Axis of parabola $x - 2 = 0$

$$\Rightarrow x = 2$$

For directrix, $y + 5 = -1$

$$\Rightarrow y + 6 = 0$$

And for focus, $y + 5 = 1$ and $x - 2 = 0$,

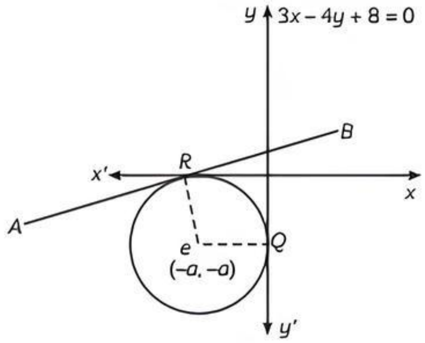
$$\Rightarrow y = -4$$

$$\Rightarrow x = 2$$

$\therefore$ Focus = (2, − 4), axis of parabola is x = 2, equation of directrix is $y + 6 = 0$ and co-ordinates of vertex are (2, −5).

31. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant. [NCERT Exemplar]

Ans. Let a be the radius of the circle. Then, the coordinates of the circle are $(-a, -a)$. Now, perpendicular distance from C to the line AB = Radius of the circle.



$$d = \left|\frac{-3a+4a+8}{\sqrt{9+16}}\right| = \left|\frac{a+8}{5}\right|$$

$$\alpha = \left(\frac{a+8}{5}\right)$$

$$\alpha = \frac{a+8}{5}$$

$$\Rightarrow 5a = a + 8$$

$$\Rightarrow 4a = 8$$

$$\Rightarrow a = 2$$

Coordinates of the centre of the circle = (−2, −2).

So, the equation of circle is,

$$(x+2)^2 + (y+2)^2 = 2^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 + 4y + 4 = 4$$

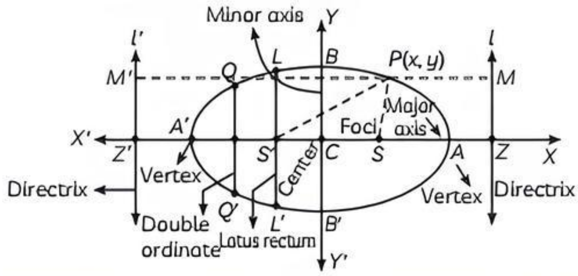
$$\Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$$

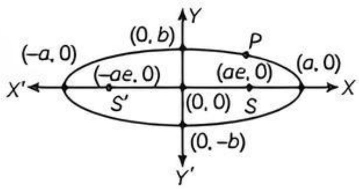
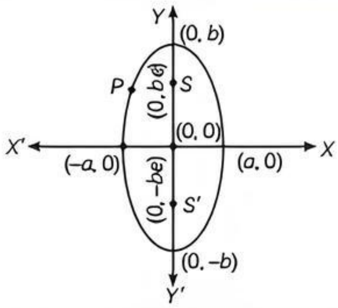
ELLIPSE AND HYPERBOLA 2

TOPIC 1

ELLIPSE

An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci of the ellipse.



Standard equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a<b$
Graph		
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x=\pm\left(\frac{a}{e}\right)$	$y=\pm\left(\frac{b}{e}\right)$
Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latus rectum	$\left(\pm ae, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{a^2}{b}, \pm be\right)$
Focal distance or radii	$\lvert SP\rvert=(a-ex_1)$ and $\lvert S'P\rvert=(a+ex_1)$	$\lvert SP\rvert=(b-ey_1)$ and $\lvert S'P\rvert=(b+ey_1)$
Sum of focal radii = | SP | + | SP |	$2a$	$2b$
Distance between foci	$2ae$	$2be$

Caution

- The constant which is the sum of the distances of a point on the ellipse from the two fixed points is always greater than the distance between the two fixed points.

Important

- If length of major axis is 2a, length of minor axis is 2b and distance between the foci is 2c, then $a^2 = b^2 + c^2$.
- Ellipse is symmetric w.r.t. both the coordinate axes.
- The foci always lie on the major axis.
- If length of semi-major axis and semi-minor axis are equal then the ellipse become circle.

Relationship between Semi-Major Axis, Semi-minor Axis and the Distance of the Focus from the Centre of the Ellipse.

For any ellipse, the semi-major axis is defined as one-half the sum of the perihelion and the aphelion, the semi-major axis is the distance from the origin to either side of the ellipse along the x-axis, or just one-half the longest axis (called the major axis).

Special Cases of an Ellipse

A circle is a special case of an ellipse, with the same radius for all points, where a^2 is always greater than b^2.

If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a^2 > b^2$ ellipse is symmetrical about x-axis.

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ where $a^2 > b^2$ ellipse is symmetrical about y-axis.

Eccentricity

The eccentricity of an ellipse is, most simply, the ratio of the distance c between the centre of the ellipse and each focus to the length of the semi-major axis a.

If the distance of the focus from the centre of the ellipse is 'c' and the distance of the end of the ellipse from the centre is 'a', then eccentricity $e=\frac{c}{a}$.

Standard Equations of an Ellipse

When the centre of the ellipse is at the origin (0, 0) and the foci are on the x-axis and y-axis, then we can easily derive the ellipse equation. The equation of the ellipse is given by,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Standard forms of the equation of an ellipse with Centre (h, k) The standard form of the equation of an ellipse with centre (h, k) and major axis parallel to the x-axis is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1.$

Latus Rectum

The chord of the ellipse through its one focus and perpendicular to the major axis (or parallel to the directrix) is called the latus rectum of the ellipse. It is a double ordinate passing through the focus. The latus rectum of an ellipse is a line passing through the foci of the ellipse and is drawn perpendicular to the transverse axis of the ellipse. The latus rectum of the ellipse is also the focal chord, which is parallel to the directrix of the ellipse. The ellipse has two foci and hence the ellipse has two latus rectums.

If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the length of latus ractum is $\frac{2b^2}{a}$.

If $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ the length of latus ractum is $\frac{2a^2}{b}$.

Example 2.1: Finds the equation for the ellipse that satisfies the given conditions: vertices (± 6, 0), foci (± 4, 0). **[NCERT]**

Ans. Vertices (±6, 0) and foci (±4, 0).

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $a = 6$, $c = 4$

It is known that $a^2 = b^2 + c^2$

$\therefore \quad 6^2 = b^2 + 4^2$

$\Rightarrow \quad 36 = b^2 + 16$

$\Rightarrow \quad b^2 = 36 - 16$

$\Rightarrow \quad b = \sqrt{20}$

Thus, the equation of the ellipse is

$$\frac{x^2}{6^2}+\frac{y^2}{(\sqrt{20})^2}=1 \text{ or } \frac{x^2}{36}+\frac{y^2}{20}=1$$

Example 2.2: Find the equation for the ellipse that satisfies the given conditions: vertices (±5, 0), foci (±4, 0). **[NCERT]**

Ans. Vertices (± 5, 0), foci (± 4, 0)

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where a is the semi-major axis.

Accordingly, $a = 5$, $c = 4$

It is known that $a^2 = b^2 + c^2$

$\therefore \quad 5^2 = b^2 + 4^2$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$

or $\frac{x^2}{25}+\frac{y^2}{9}=1$

Example 2.3: Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ [NCERT]

Ans. The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis. On comparing the given equation with

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 4$ and $b = 3$.

$\therefore \quad c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $(\pm\sqrt{7},0)$

The coordinates of the vertices are $(\pm 4, 0)$

Length of major axis = $2a = 8$

Length of minor axis = $2b = 6$

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Example 2.4: Find the coordinates of the focii, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ [NCERT]

Ans. The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis. On comparing the given equation with

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

$\therefore \quad c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

we obtain $a = 6$ and $b = 4$.

Therefore,

The coordinates of the foci are $(2\sqrt{5},0)$ and $(-2\sqrt{5},0)$

The coordinates of the vertices are (6, 0) and (–6, 0)

Length of major axis = $2a = 12$

Length of minor axis = $2b = 8$

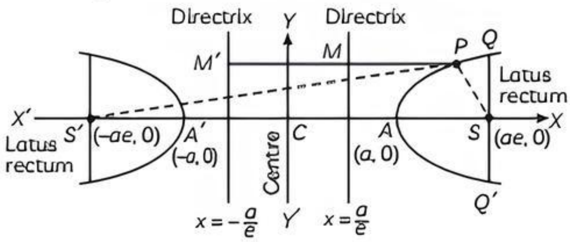
Eccentricity, $e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

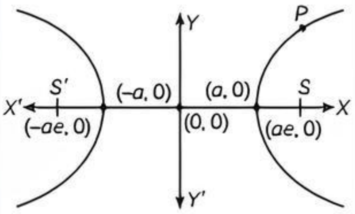
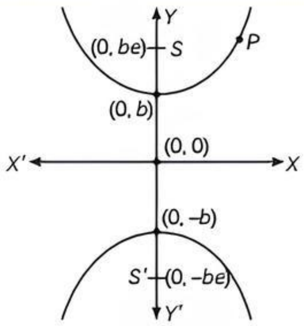
Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 16}{6}=\frac{16}{3}$

TOPIC 2

HYPERBOLA

A hyperbola is the set of all points in a plane, the difference of whose distance from two fixed points is constant.



Fundamental terms	Hyperbola	Conjugate Hyperbola
Standard equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$
Graph		
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	$(\pm a, 0)$	$(0, \pm b)$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x=\pm\left(\frac{a}{e}\right)$	$y=\pm\left(\frac{b}{e}\right)$
Eccentricity	$e=\sqrt{1+\frac{b^2}{a^2}}$	$e=\sqrt{1+\frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latus rectum	$\left(\pm ae, \pm\frac{b^2}{a}\right)$	$\left(\pm\frac{a^2}{b}, \pm be\right)$
Difference of focal radii = $\lvert SP \rvert - \lvert S'P \rvert$	$2a$	$2b$
Distance between foci	$2ae$	$2be$

Standard Equation of Hyperbola

A hyperbola is the locus of all those points in a plane such that the difference in their distances from two fixed points in the plane is a constant.

When both the foci are joined with the help of a line segment then the mid-point of this line segment joining the foci is known as the centre, O represents the centre of an ellipse in the figure given above. The line segment passing through both foci is the transverse axis of the hyperbola. The line segment perpendicular to the transverse axis and passing through the centre represents the conjugate axis of the hyperbola.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$$

These are the standard equations of hyperbola.

Latus Rectum

The line segments perpendicular to the transverse axis through any of the foci such that their end points lie on the hyperbola are defined as the latus rectum of a hyperbola.

Length of Latus Rectum of Hyperbola

Latus rectum of a hyperbola is defined analogously as in the case of parabola and ellipse.

The ends of the latus rectum of a hyperbola are $\left(\pm ae, \pm\frac{b^2}{a^2}\right)$, and the length of the latus rectum is $\frac{2b^2}{a}$.

Important

- A hyperbola in which $a = b$ is called a rectangular hyperbola.
- Hyperbola is symmetric w.r.t. both the axes.
- The foci are always on the transverse axis.

Example 2.5: Find the equation of the hyperbola satisfy the given conditions: vertices ($\pm$ 2, 0), foci ($\pm$ 3, 0). [NCERT]

Ans. Vertices ($\pm$2, 0), foci ($\pm$3, 0)

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Since the vertices are ($\pm$2, 0), $a = 2$

Since the foci are ($\pm$3, 0), $c = 3$.

We know that $a^2 + b^2 = c^2$

$\therefore \quad 2^2 + b^2 = 3^2$

$b^2 = 9 - 4 = 5$

Thus, the equation of the hyperbola is $\frac{x^2}{4}-\frac{y^2}{5}=1$

Example 2.6: Find the equation of the hyperbola satisfying the given conditions: vertices (0, $\pm$5), foci (0, $\pm$8). [NCERT]

Ans. Vertices (0, $\pm$5), foci (0, $\pm$8)

Here, the vertices are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

Since the vertices are (0, $\pm$5), $a = 5$.

Since the foci are (0, $\pm$8), $c = 8$.

We know that, $a^2 + b^2 = c^2$.

$\therefore \quad 5^2 + b^2 = 8^2$

$b^2 = 64 - 25 = 39$

Thus, the equation of the hyperbola is

$$\frac{y^2}{25}-\frac{x^2}{39}=1.$$

Example 2.7: Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$ [NCERT]

Ans. The given equation is $49y^2 - 16x^2 = 784$

It can be written as

$49y^2 - 16x^2 = 784$

Or, $\frac{y^2}{16}-\frac{x^2}{49}=1$

Or, $\frac{y^2}{4^2}-\frac{x^2}{7^2}=1$ ...(i)

On comparing equation (i) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain $a = 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$

$\therefore \quad c^2 = 16 + 49 = 65$

$\Rightarrow \quad c = \sqrt{65}$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{65})$.

The coordinates of the vertices are (0, $\pm$ 4).

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{65}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 49}{4}=\frac{49}{2}$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ meets the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 - 10b + 25$, then the value of b does not satisfy:

(a) $(-\infty, 4)$ (b) [4, 6]

(c) $(6, \infty)$ (d) none of these

Ans. *(b)* [4, 6]

Explanation: Now according to condition $a > 1$

$\Rightarrow \quad b^2 - 10b + 25 > 1$

$\Rightarrow \quad b^2 - 10b + 24 > 0$

$\Rightarrow \quad (b - 4)(b - 6) > 0$

$\Rightarrow \quad b < 4$ or $b > 6$

$\Rightarrow \quad b \notin [4, 6]$

2. The sum of the distance of any point on the ellipse $3x^2 + 4y^2 = 24$ from its foci is:

(a) $4\sqrt{2}$ (b) 8

(c) $16\sqrt{2}$ (d) $2\sqrt{2}$

Ans. *(a)* $4\sqrt{2}$

Explanation: Given equation of ellipse is,

$3x^2 + 4y^2 = 24$

$$\frac{3x^2}{24}+\frac{4y^2}{24}=1$$

$$\frac{x^2}{8}+\frac{y^2}{6}=1$$

On comparing it with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

$\Rightarrow \quad a^2 = 8$

$\Rightarrow \quad a = 2\sqrt{2}$

The sum of focal distances of a point of the ellipse = length of major axis = $2a = 4\sqrt{2}$

3. **Find the coordinates of the foci and eccentricity of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$.**

(a) $(0, \pm 4), \frac{4}{5}$ (b) $(\pm 4, 0), \frac{4}{5}$

(c) $(0, \pm 4), \frac{4}{3}$ (d) $(0, \pm 2), \frac{4}{5}$

Ans. *(b)* $(\pm 4, 0), \frac{4}{5}$

Explanation: The given equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

On comparing it with $\frac{x^2}{a^2}+\frac{y^2}{b^2}$, we get

$a = 5$ and $b = 3$

$$\therefore \quad e=\sqrt{1-\frac{b^2}{a^2}}=\frac{4}{5}$$

Therefore, the coordinates of the foci are, $(\pm ae, 0)$ i.e., $(\pm 4, 0)$

4. **If the length of the major axis of an ellipse is $\frac{17}{8}$ times the length of the minor axis, then the eccentricity of the ellipse is:**

(a) $\frac{8}{17}$ (b) $\frac{15}{17}$

(c) $\frac{9}{17}$ (d) $\frac{2\sqrt{2}}{17}$

Ans. *(b)* $\frac{15}{17}$

Explanation: Length of major axis

$$= \frac{17}{8} \times \text{length of minor axis}$$

$\Rightarrow \quad 2a=\frac{17}{8}\times 2b$

$\Rightarrow \quad \frac{b}{a}=\frac{8}{17}$

$\Rightarrow \quad e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$

5. **Coordinates of focii of $9x^2 - 16y^2 = 144$ are:**

(a) (3, 0); (–3, 0) (b) (4, 0); (–4, 0)

(c) (5, 0); (–5, 0) (d) (12, 0); (–12, 0)

[Diksha]

Ans. *(c) (5, 0); (–5, 0)*

Explanation: Given, $9x^2 - 16y^2 = 144$

$$\frac{9x^2}{144}-\frac{16y^2}{144}=1$$

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

$$a^2 = 16$$

$$b^2 = 9$$

$$c = \pm 5$$

So coordinate of focii is $(\pm c, 0)$

i.e., $(\pm 5, 0)$

6. **The equation $\frac{x^2}{1-r}-\frac{y^2}{1+r}=1$, $|r| < 1$ represents a/an:**

(a) ellipse (b) hyperbola

(c) circle (d) none of these

Ans. *(b) hyperbola*

Explanation: Since, $|r| < 1$, $1 - r$ and $1 + r$ are both positive.

So, we put $1 - r = a^2$ and $1 + r = b^2$.

Then the given equation becomes, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which represents a hyperbola.

7. **The eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and $(3\sqrt{2}, 2)$ is:**

(a) $\frac{1}{\sqrt{13}}$ (b) $\sqrt{13}$

(c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{13}}{3}$

Ans. *(d)* $\frac{\sqrt{13}}{3}$

Explanation: Given that the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through the points (3, 0) and $(3\sqrt{2}, 2)$, so we get $a^2 = 9$ and $b^2 = 4$.

Again, we know that $b^2 = a^2 (e^2 - 1)$. This gives

$$4 = 9(e^2 - 1)$$

$$\Rightarrow \quad e^2 = \frac{13}{9}$$

$$\Rightarrow \quad e = \frac{\sqrt{13}}{3}$$

8. If e_1 is the eccentricity of the conic $9x^2 + 4y^2 = 36$ and e_2 is the eccentricity of the conic, $9x^2 - 4y^2 = 36$ then which of the following is true?

(a) $e_1^2 + e_2^2 = 2$ (b) $3 < e_1^2 + e_2^2 < 4$

(c) $e_1^2 + e_2^2 > 4$ (d) None of these

Ans. *(b) $3 < e_1^2 + e_2^2 < 4$*

Explanation: Given,

$$9x^2 + 4y^2 = 36$$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$e_1^2 = \frac{9-4}{9}$$

$$= \frac{5}{9}$$

and given,

$$9x^2 - 4y^2 = 36$$

$$\frac{9x^2}{36} - \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\therefore \quad e_2^2 = \frac{4+9}{4} = \frac{13}{4}$$

Now $e_1^2 + e_2^2 = \left(\frac{5}{9}\right) + \left(\frac{13}{4}\right) = \frac{137}{36} > 3$ but < 4

$\therefore \quad 3 < e_1^2 + e_2^2 < 4$

9. The eccentricity of the hyperbola $x^2 - y^2 = 2004$ is:

(a) $\sqrt{3}$ (b) 2

(c) $2\sqrt{2}$ (d) $\sqrt{2}$

Ans. *(d) $\sqrt{2}$*

Explanation: Given equation of hyperbola is,

$$x^2 - y^2 = 2004$$

Or $\frac{x^2}{(\sqrt{2004})^2} - \frac{y^2}{(\sqrt{2004})^2} = 1$

On comparing it with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$\therefore \quad a = \sqrt{2004}$ and $b = \sqrt{2004}$

Using, $b^2 = a^2 (e^2 - 1)$

or, $e^2 - 1 = 1$

or, $e^2 = 2$ or, $e = \sqrt{2}$

10. Eccentricity of $9x^2 - 16y^2 = 144$ is:

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$ [Diksha]

Ans. *(d) $\frac{5}{4}$*

Explanation: Given, equation of circle is,

$$9x^2 - 16y^2 = 144$$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$a^2 = 16$$

$$b^2 = 9$$

$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 9$$

$$c^2 = 25$$

$$c = 5$$

$$e = \frac{c}{a}$$

$$e = \frac{5}{4}$$

11. The equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directrices is:

(a) $\frac{x^2}{36} + \frac{y^2}{11} = 4$ (b) $\frac{x^2}{6} + \frac{y^2}{11} = 1$

(c) $\frac{x^2}{36} + \frac{y^2}{11} = 1$ (d) $\frac{x^2}{36} + \frac{y^2}{4} = 1$

Ans. *(c) $\frac{x^2}{36} + \frac{y^2}{11} = 1$*

Explanation: We have $ae = 5$, $\frac{a}{e} = \frac{36}{5}$, which gives, $a = 6$ and $e = \frac{5}{6}$.

Now, $b = a\sqrt{1 - e^2} = 6\sqrt{1 - \frac{25}{36}} = \sqrt{11}$

Thus, the equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{11} = 1$.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

12. Assertion (A): The sum of focal distances of a point on the ellipse $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ is 4.

Reason (R): The equation $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ can be expressed as $9(x-1)^2 + 4(y-3)^2 = 36$.

Ans. *(d) (A) is false but (R) is true.*

Explanation: We have,

$$9x^2 + 4y^2 - 18x - 24y + 9 = 0$$

$$9x^2 - 18x + 9 + 24y = 0$$

$$\Rightarrow \quad 9(x-1)^2 + 4(y-3)^2 = 36$$

$$\Rightarrow \quad \frac{(x-1)^2}{2^2} + \frac{(y-3)^2}{3^2} = 1$$

Here, $b > a$

$\therefore$ Sum of focal distance of a point is $2b = 6$.

13. Assertion (A): The length of major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and, 10, respectively.

Reason (R): The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y-3)^2 = 45$.

Ans. *(d) (A) is false but (R) is true.*

Explanation: We have,

$$5x^2 + 9y^2 - 54y + 36 = 0$$

$$\Rightarrow \quad 5x^2 + 9(y-3)^2 = 45$$

$$\Rightarrow \quad \frac{x^2}{3^2} + \frac{(y-3)^2}{(\sqrt{5})^2} = 1$$

$\therefore$ Length of major axis $= 2 \times 3 = 6$

And length of minor axis $= 2 \times \sqrt{5} = 2\sqrt{5}$

14. If the distance of foci and vertex of hyperbola from the centre are c and a respectively, then

Assertion (A): Eccentricity is always less than 1.

Reason (R): Focii are at a distance of ae from the centre.

Ans. *(d) (A) is false but (R) is true.*

Explanation: Since, $c \geq a$, the eccentricity is near less than one. In terms of the eccentricity, the focii are at a distance of ae from the centre.

15. Assertion (A): The foci of the hyperbola $9x^2 - 16y^2 = 144$ is $(\pm 5, 0)$.

Reason (R): The formula to find the focii of a parabola is $c^2 = a^2 + b^2$.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Given,

$$9x^2 - 16y^2 = 144$$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Here, $a^2 = 16$ and $b^2 = 9$

So, $c^2 = 16 + 9 = 25$

$c = \pm 5$,

Hence, coordinate of foci is $(\pm 5, 0)$

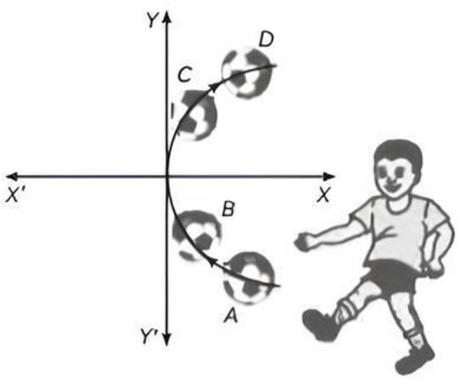
To find the coordinate of foci we have to use $c^2 = a^2 + b^2$.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

16. *Arun was playing a football match. When he kicked the football, the path formed by the football from ground level is parabolic, which is shown in the following graph. Consider the coordinates of point A as (3, –2).*



(A) The equation of path formed by the football is:

(a) $y^2 = x + 1$ (b) $3x^2 = 4y$

(c) $3y^2 = 4x$ (d) $x^2 = y - 1$

(B) The equation of directrix of path formed by football is:

(a) $x - \frac{4}{3} = 0$ (b) $x + \frac{2}{3} = 0$

(c) $x + 3 = 0$ (d) $x + \frac{1}{3} = 0$

(C) The extremities of latus rectum of given curve are:

(a) $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$ (b) $\left(\frac{2}{3}, \pm\frac{1}{3}\right)$

(c) $\left(\pm\frac{1}{3}, 1\right)$ (d) $\left(\pm\frac{1}{3}, \frac{4}{3}\right)$

(D) The length of latus rectum of a given curve is:

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$

(c) 3 (d) $\frac{4}{3}$

(E) The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is:

(a) $x = 1$ (b) $x = -1$

(c) $x = \frac{3}{2}$ (d) $x = -\frac{3}{2}$

Ans. (A) (c) $3y^2 = 4x$

Explanation: The path formed by football is in the shape of parabola. We know that general equation of parabola is $y^2 = 4ax$.

Since, it passes through (3, −2)

$\therefore \quad (-2)^2 = 4 \times a \times 3$

$\Rightarrow \quad a = \frac{1}{3}$

$\Rightarrow$ Hence, required equation of path formed by football is $y^2 = \frac{4x}{3}$

$\Rightarrow \quad 3y^2 = 4x$

(B) (d) $x + \frac{1}{3} = 0$

Explanation: Since, $a = \frac{1}{3}$. Therefore, the equation of its directrix is $x + \frac{1}{3} = 0$.

(C) (a) $\left(\frac{1}{3}, \pm\frac{2}{3}\right)$

Explanation: The extremities of latus rectum are $(a, \pm 2a) = \left(\frac{1}{3}, \pm\frac{2}{3}\right)$

(D) (d) $\frac{4}{3}$

Explanation: The length of latus rectum

$$= 4a = 4 \times \frac{1}{3} = \frac{4}{3}$$

(E) (c) $x = \frac{3}{2}$

Explanation: Given equation:

$$y^2 + 4y \;\; 4x \;\; 2 = 0$$

Rearranging the equation, we get

$$(y + 2)^2 = -4x + 2$$

$$(y + 2)^2 = -4\left(x - \left(\frac{1}{2}\right)\right)$$

Let $\quad y = y + 2$ and $x - \left(\frac{1}{2}\right)$

So, $\quad y^2 = -4X \quad$ ...(i)

Hence, equation (i) is of the form

$$y^2 = -4ax \quad \text{...(ii)}$$

By comparing (i) and (ii), we get $a = 1$.

We know that equation of directrix is $x = a$

Now, substitute $a = 1$ and $x = x = x - \left(\frac{1}{2}\right)$ in the directrix equation.

$$x - \left(\frac{1}{2}\right) = 1$$

$$x = 1 + \left(\frac{1}{2}\right) = \frac{3}{2}.$$

Therefore, the equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is $\frac{3}{2}$.

17. *Karan, the student of class XI was studying in his house. He felt hungry and found that his mother was not at home. So, he went to the nearby shop and purchased a packet of chips. While eating the chips, he observed that one piece of the chips is in the shape of hyperbola. Consider the vertices of the hyperbola at (± 5, 0) and focii at (± 7, 0).*

(A) Find the equation of hyperbolic curve and the length of conjugate axis formed by a given piece of chip.

(B) Find the eccentricity and length of latus rectum of the hyperbolic curve formed by a given piece of chips.

(C) Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola $3x^2 - y^2 = 4$.

Ans. (A) We have, $a = 5$ and $ae = 7$

Now, $b^2 = a^2e^2 - a^2$

$= 49 - 25$

$= 24$

So, equation is $\frac{x^2}{25} - \frac{y^2}{24} = 1$

Length of conjugate axis $= 2b$

$= 2 \times 2\sqrt{6}$

$= 4\sqrt{6}$

(B) Eccentricity,

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$$

Length of latus-rectum $= \frac{2b^2}{a} = \frac{48}{5} = 9.6$

(C) Given,

The equation $3x^2 - y^2 = 4$

The equation can be expressed as:

$$\frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{(2)^2} = 1$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = \frac{2}{\sqrt{3}}$ and $b = 2$

Eccentricity is given by:

$$e = \sqrt{1+\frac{b^2}{a^2}}$$

$$= \sqrt{1+\frac{4}{\frac{4}{3}}}$$

$$= \sqrt{1+3}$$

$$= \sqrt{4}$$

$$= 2$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$(\pm ae, 0) = \pm\left(\frac{2}{\sqrt{3}}\right)(2) = \pm\frac{4}{\sqrt{3}}$$

$$(\pm ae, 0) = \left(\pm\frac{4}{\sqrt{3}}, 0\right)$$

The equation of directrices is given as:

$$x = \pm\frac{a}{e}$$

$\Rightarrow$ $$x = \pm\frac{\frac{2}{\sqrt{3}}}{2}$$

$\Rightarrow$ $$x = \pm\frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\sqrt{3}x = \pm 1$$

The length of the latus-rectum is given as:

$$\frac{2b^2}{a} = \frac{2(4)}{\left[\frac{2}{\sqrt{3}}\right]}$$

$$= 4\sqrt{3}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

18. If the latus rectum of an ellipse is equal to half of the minor axis, then find its eccentricity. [NCERT Exemplar]

Ans. Consider, the equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is given that,

length of latus rectum = half of the minor axis

$\Rightarrow$ $$\frac{2b^2}{a} = b$$

$\Rightarrow$ $a = 2b$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $b^2 = 4b^2(1 - e^2)$

$\Rightarrow$ $$1 - e^2 = \frac{1}{4}$$

$\Rightarrow$ $$e^2 = \frac{3}{4}$$

$\therefore$ $$e = \frac{\sqrt{3}}{2}$$

19. Find the equation of ellipse whose focus is (0, 17) and vertex is (0, 300).

Ans. Let the equation will be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, focus $(0, c) = (0, 17)$ and vertex

$$(0, a) = (0, 300)$$
$$c = 17$$
$$c^2 = a^2 - b^2$$
$$17^2 = a^2 - b^2$$
$$289 - 300 = -b^2$$
$$-11 = -b^2$$
$$b^2 = 11$$

Hence, the equation of the ellipse is,

$$\frac{x^2}{11} + \frac{y^2}{300} = 1$$

20. Given the ellipse with equation,

$$9x^2 + 25y^2 = 225,$$

find the eccentricity and foci.

[NCERT Exemplar]

Ans. Given equation of ellipse, $9x^2 + 25y^2 = 225$

Or $\frac{x^2}{25} + \frac{y^2}{9} = 1$

On comparing it with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$$a = 5 \text{ and } b = 3$$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow$ $9 = 25(1 - e^2)$

$\Rightarrow$ $\frac{9}{25} = 1 - e^2$

$\Rightarrow$ $e^2 = 1 - \frac{9}{25}$

$= \frac{16}{25}$

$\therefore$ $e = \frac{4}{5}$

$$\text{Foci} = (\pm ae, 0) = \left(\pm 5 \times \left(\frac{4}{5}\right) 0\right)$$
$$= (\pm 4, 0)$$

21. Find the equation of hyperbola whose length of transverse axis 10 and conjugate axis is 16.

Ans. Length of transverse axis $= 2a$

$$= 2a = 10$$
$$a = 5$$

Length of conjugate axis $= 2b$

$$= 2b = 16$$
$$b = 8$$

Hence, required equation of hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{64} = 1$$

22. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

[Delhi Gov. QB 2022]

Ans. Given, $e = 0$

$b^2 = a^2(1 - e^2)$ or $a^2 = b^2(1 - e^2)$

$b^2 = a^2$ $\qquad$ $a^2 = b^2$

$\therefore$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$
$$x^2 + y^2 = a^2$$

Which is the equation of a circle which centre (0, 0) and radius = a.

Hence, proved.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

23. If the eccentricity of the hyperbola is $\sqrt{2}$. Then, find the general equation of hyperbola.

Ans. Given, eccentricity $e = \sqrt{2}$

We know that

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$
$$\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$$

$\Rightarrow$ $\sqrt{a^2 + b^2} = \sqrt{2}a$

$\Rightarrow$ $a^2 + b^2 = 2a^2$

[Squaring on both sides]

$\Rightarrow$ $a^2 = b^2$

$\therefore$ Equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \text{ or } \frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$$

$\Rightarrow$ $x^2 - y^2 = a^2$

$\Rightarrow$ $y^2 - x^2 = a^2$

24. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of the hyperbola. [NCERT Exemplar]

Ans. Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Foci are $(\pm ae, 0)$

Distance between foci = $2ae = 16$ [Given]

$\Rightarrow \quad ae = 8$

Also, $\quad e = \sqrt{2}$ [Given]

$\Rightarrow \quad a \times \sqrt{2} = 8$

$\Rightarrow \quad a = \dfrac{8}{\sqrt{2}}$

$\therefore \quad a = 4\sqrt{2}$

We know that, $b^2 = a^2 (e^2 - 1)$

$\Rightarrow \quad b^2 = (4\sqrt{2})^2\left[(\sqrt{2})^2 - 1\right]$

$= 16 \times 2(2 - 1) = 32$

So, the equation of hyperbola is: $\dfrac{x^2}{32} - \dfrac{y^2}{32} = 1$ or $x^2 - y^2 = 32$.

25. Find the eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci.

[Delhi Gov. QB 2022]

Ans. Given that the length of the latus rectum is 8 and length of the conjugate axis is equal to half the distance between the foci.

$\Rightarrow \quad \dfrac{2b^2}{a} = 8$ and $2b = \dfrac{1}{2}(2ae)$

$\therefore \quad \dfrac{2}{a}\left(\dfrac{ae}{2}\right)^2 = 8$

$\Rightarrow \quad ae^2 = 16$ ...(i)

We have $\quad \dfrac{2b^2}{a} = 8$

$\Rightarrow \quad b^2 = 4a$

$\Rightarrow \quad a^2(e^2 - 1) = 4a$

$\Rightarrow \quad ae^2 - a = 4$

$\Rightarrow \quad 16 - a = 4$ [by (i)]

$\Rightarrow \quad a = 12$

Substitute $a = 12$ in (i)

$\Rightarrow \quad 12e^2 = 16$

$\Rightarrow \quad e^2 = \dfrac{4}{3}$

$\therefore \quad e = \dfrac{2}{\sqrt{3}}$

26. Find the coordinate of foci, vertex, length of major axis of the ellipse

$$\frac{x^2}{90} + \frac{y^2}{9} = 1.$$

Ans. From the equation $\dfrac{x^2}{90} + \dfrac{y^2}{9} = 1$

On comparing with $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ we get

$a^2 = 90$

$\Rightarrow \quad a = \sqrt{90}$

$\Rightarrow \quad a = 3\sqrt{10}$

$b^2 = 9$

$\Rightarrow \quad b = 3$

Now $\quad c = \sqrt{a^2 - b^2}$

$= \sqrt{90 - 9} = 9$

Coordinate of foci are $(\pm c, 0) = (\pm 9, 0)$

Vertex, $\quad (\pm a, 0) = (\pm\sqrt{90}, 0)$

Length of major axis = $2a$

$= 2 \times 3\sqrt{10}$

$= 6\sqrt{10}$

27. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then obtain the equation of a hyperbola.

[Delhi Gov. QB 2022]

Ans. The distance between the foci is $2ae$

$\therefore \quad 2ae = 16$

$\Rightarrow \quad ae = 8$

$e = \sqrt{2}$

$\therefore \quad a\sqrt{2} = 8$

$\Rightarrow \quad a = 4\sqrt{2}$

Also, $\quad b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad b^2 = 32(2 - 1)$

$\Rightarrow \quad b^2 = 32$

Standard form of the hyperbola is given by

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$x^2 - y^2 = 32$$

28. Find the equation of ellipse whose eccentricity is $\dfrac{2}{3}$, latus rectum is 5 and the centre is (0, 0).

[NCERT Exemplar]

Ans. Let equation of the ellipse be, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$

Given that, $e = \dfrac{2}{3}$ and latus rectum = 5

$\therefore \quad \dfrac{2b^2}{a} = 5$

$b^2 = \dfrac{5a}{2}$

We know that, $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad \frac{5a}{2} = a^2\left(1 - \frac{4}{9}\right)$

$\Rightarrow \quad \frac{5}{2} = \frac{5a}{9}$

$\Rightarrow \quad a = \frac{9}{2}$

$\therefore \quad b^2 = \frac{5\times 9}{2\times 2} = \frac{45}{4}$

So, the required equation of the ellipse is

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1.$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

29. Find the equation of hyperbola for which the length of the transverse axis along x-axis with centre at origin of a hyperbola is 7 and it passes through the point (5, −2).

Ans. Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ represent the hyperbola.

According to the given condition, the length of transverse axis i.e., $2a = 7$

$\Rightarrow \quad a = \frac{7}{2}$

Also, the point (5, −2) lies on the hyperbola, so, we have

$$\frac{(5)^2}{\left(\frac{7}{2}\right)^2} - \frac{(-2)^2}{b^2} = 1$$

$$\frac{4}{49}(25) - \frac{4}{b^2} = 1$$

which gives

$\Rightarrow \quad \frac{100}{49} - \frac{4}{b^2} = 1$

$\frac{100}{49} - 1 = \frac{4}{b^2}$

$\Rightarrow \quad \frac{4}{b^2} = \frac{51}{49}$

$b^2 = \frac{196}{51}$

Hence, equation of hyperbola is

$$\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1.$$

30. Write the equation of directrix of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ such that $a > b$.

Ans. The given equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$$

$\therefore$ Equation of the directrices $x = \pm\frac{a}{e}$.

The equation of directrices of the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is also $x = \pm\frac{a}{e}$.

31. Check whether the line $2x + 3y = 12$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 2$ at the point (3, 2). **[NCERT Exemplar]**

Ans. Given line is $2x + 3y = 12$ and the ellipse is $4x^2 + 9y^2 = 72$

Solving line and ellipse, we get

$(12 - 3y)^2 + 9y^2 = 72$

$\Rightarrow \quad (4 - y)^2 + y^2 = 8$

$\Rightarrow \quad 16 - 8y + y^2 + y^2 = 8$

$\Rightarrow \quad 2y^2 - 8y + 8 = 0$

$\Rightarrow \quad y^2 - 4y + 4 = 0$

$\Rightarrow \quad (y - 2)^2 = 0$

$\Rightarrow \quad y = 2$

$\Rightarrow \quad 2x = 12 - 3(2)$

$\Rightarrow \quad 2x = 6$

[from the equation of line]

$\Rightarrow \quad x = 3$

So, point of contact is (3, 2)

Hence, verified.

32. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$ then find the value of m. **[NCERT Exemplar]**

Ans. Given that, line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$.

Solving line with parabola, we have

$(mx + 1)^2 = 4x$

$\Rightarrow \quad m^2x^2 + 2mx + 1 = 4x$

$\Rightarrow \quad m^2x^2 + x(2m - 4) + 1 = 0$

Since, the line touches the parabola, above equation must have equal roots.

$\therefore$ Discriminant, $D = 0$

$\Rightarrow \quad (2m - 4)^2 - 4m^2 = 0$

$\Rightarrow \quad 4m^2 - 16m + 16 - 4m^2 = 0$

$\Rightarrow \quad 16m = 16$

$\therefore \quad m = 1$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

33. Find the equation for the ellipse that satisfies the given condition: major axis on the x-axis and passes through the points (2, 1) and (3, 2).

Ans. Let the equation of the ellipse with major axis on x-axis be,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1 \qquad (a > b) \text{ ...(i)}$$

Since the ellipse passes through (2, 1) and (3, 2).

$$\therefore \quad \frac{4}{a^2}+\frac{1}{b^2} = 1 \qquad \text{...(ii)}$$

$$\text{And} \quad \frac{9}{a^2}+\frac{4}{b^2} = 1 \qquad \text{...(iii)}$$

On solving eq. (ii) and (iii), we obtain

$$\Rightarrow \quad a^2 = \frac{5}{3} \text{ and } b^2 = \frac{5}{7}$$

$\therefore$ Thus, the equation of an ellipse is $\frac{3x^2}{5}+\frac{7y^2}{5}=1$, which is the required equation of the ellipse.

34. Find the equation of the ellipse centre is at origin and the major axis, which passes through the points (–3, 1) and (2, –2). [NCERT Exemplar]

Ans. As origin is the centre of ellipse, so $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the equation of ellipse.

Since, this ellipse passes through (–3, 1) and (2, – 2) so,

$$\frac{9}{a^2}+\frac{1}{b^2}=1 \text{ and } \frac{4}{a^2}+\frac{4}{b^2}=1$$

$$\Rightarrow \frac{9}{a^2}+\frac{1}{b^2}=1 \text{ and } \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$$

which gives $\frac{8}{a^2}=\frac{3}{4}$

$$\Rightarrow \quad a^2=\frac{32}{3}$$

$$\text{And} \quad \frac{1}{b^2}=1-\frac{9}{a^2} =1-\frac{9}{\frac{32}{3}} =1-\frac{27}{32}=\frac{5}{32}$$

$$\Rightarrow \quad b^2=\frac{32}{5}$$

Hence, equation of ellipse is

$$\frac{x^2}{\frac{32}{3}}+\frac{y^2}{\frac{32}{5}}=1$$

$$\Rightarrow \quad 3x^2 + 5y^2 = 32.$$

35. Find the equation of the hyperbola whose conjugate axis is 6 and distance between the foci is 12.

Ans. Let the equation of the ellipse be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad [a > b] \text{ ...(i)}$$

Length of conjugate axis = 6

$\Rightarrow \quad 2b = 6$

$\Rightarrow \quad b = 3$

And distance between the foci = 12

$\Rightarrow \quad 2c = 12$

$\Rightarrow \quad c = 6$

Since, $\quad c^2 = a^2 + b^2$

$\Rightarrow \quad a^2 = c^2 - b^2 = 36 - 9 = 27$

$\Rightarrow \quad a^2 = 27$

$$\therefore \quad \frac{x^2}{27}-\frac{y^2}{9} = 1$$

$\frac{x^2}{27}-\frac{y^2}{9}=1$, is the required equation of the hyperbola.

36. If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find latus rectum of the ellipse. [NCERT Exemplar]

Ans. Let equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$

Given that, eccentricity, $e=\frac{5}{8}$

Now, the foci of this ellipse are ($\pm$ ae, 0)

Distance between foci = 10 (Given)

$\therefore \quad 2ae = 10$

$\Rightarrow \quad ae = 5$

$\Rightarrow \quad \frac{5}{8}a = 5$

$\Rightarrow \quad a = 8$

We know that, $b^2 = a^2 (1 - e^2)$

$$\Rightarrow \quad b^2 = 64\left(1-\frac{25}{64}\right) = 64 - 25 = 39$$

$\therefore$ Length of latus rectum of ellipse

$$=\frac{2b^2}{a}=2\times\frac{39}{8}=\frac{39}{4}.$$